THE EARLY UNIVERSE ODYSSEY WITH GRAVITATIONAL WAVES

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This contribution summarizes some recent work on gravitational-wave astronomy and, especially, on the generation and detection of relic gravitational waves. We begin with a brief discussion of astrophysical sources of gravitational waves that are likely to be detected first by the coming in operation laser interferometers, such as GEO, LIGO, VIRGO. Then, we proceed to relic gravitational waves emphasizing their quantum-mechanical origin and the inevitability of their existence. Combining the theory with available observations, we discuss the prospects of direct detection of relic gravitational waves. A considerable part of the paper is devoted to comparison of relic gravitational waves with the density perturbations of quantum-mechanical origin. It is shown how the phenomenon of squeezing of quantum-mechanically generated cosmological perturbations manifests itself in the periodic structures of the metric power spectra and in the oscillatory behaviour of the CMBR multipoles C_l as a function of l. The cosmological importance of the theoretically calculated (statistical) dipole moment C_1 is stressed. The paper contains also some comments on the damage to gravitational-wave research inflicted by the "standard inflationary result". We conclude with the (now common) remarks on the great scientific importance of the continuing effort to observe relic gravitational waves, directly or indirectly.

1 Which sources of gravitational waves will be detected first ?

The comprehensive contributions of Coccia [1] and Giazotto [2] have described the outstanding experimental effort that is now going on at detecting gravitational waves. As we all know, great expectations are related with the coming in operation laser interferometers: the British-German GEO600 [3], the two American instruments LIGO [4], and the French-Italian VIRGO [5]. It is important to know in advance, and to be prepared to monitor, those sources of gravitational waves that are likely to be detected first in the ongoing and forthcoming observations. Before proceeding to the main topic of my contribution - relic gravitational waves - I would like to present a theorist's view on this issue. I will briefly report on the conclusions of a detailed study undertaken in Ref.[6]. By comparing our theoretical analysis with only 3 out of several experimental programs, we, of course, do not mean to diminish the

arXiv:gr-qc/0202072v2 31 Oct 2002

importance of work of other experimental groups.

It is common to divide possible sources of gravitational waves in three broad categories: quasi-periodic, explosive, and stochastic. We are aware of definitely existing astrophysical sources belonging to each category. To be interesting from the point of view of its detection, the source should be sufficiently powerful, should fall in the frequency band of the detector, and should occur reasonably often during the life-time of the instrument. After having analysed many possible sources, and taking all the factors into account, the authors of Ref.[6] gave their preference to compact binaries consisting of black holes and neutron stars. It is clear that in order to radiate large-intensity gravitational waves at frequencies accesible to ground-based instruments, the objects forming a pair should be massive and should orbit each other at very small separations - a few hundreds kilometers. According to the existing views, such massive objects can only be the end-products of stellar evolution - neutron stars (NS) and black holes (BH). Compact binary systems of our interest are coalescing pairs that are only tens of minutes away from the final merger, i.e., from the formation of a resulting black hole or, possibly, from another spectacular event, such as a gamma-ray burst. The important question is how many such close binaries exist in our Galaxy and at cosmological distances. This determines the event rate, that is, the number of coalescence events that are likely to occur in a given cosmological volume during, say, a 1 year of observations. The event rate is partially constrained by the existing observations of binary pulsars, but its evaluation mostly relies on the numerical modelling of diverse evolutionary tracks of massive binary stars.

The event rates are sensitive to several evolutionary parameters, one of which is the so-called kick velocity parameter w_0. This parameter enters the distribution function for velocities that may be imparted on a neutron star or a black hole at the moment of its formation. It is believed that w_0 lies in the interval $(200 - 400)km/sec$. In Fig. 1 we plot the NS+NS, NS+BH, and BH+BH merging rates as functions of w_0. This calculation has been performed for a typical spiral galaxy of the mass $10^{11}M_\odot$ and under a reasonable choice of other evolutionary parameters [7].

It follows from this graph that the NS+NS rate is expected to be at the level $3 \times 10^{-5}year^{-1}$, whereas the BH+BH rate is, at least, one order of magnitude lower, i.e., $3 \times 10^{-6}year^{-1}$. These rates for a typical galaxy, $\mathcal{R}_G$, define the rates for a given cosmological volume, $\mathcal{R}_V$, which includes many galaxies. When deriving $\mathcal{R}_V$, we have used a conservative estimate for the
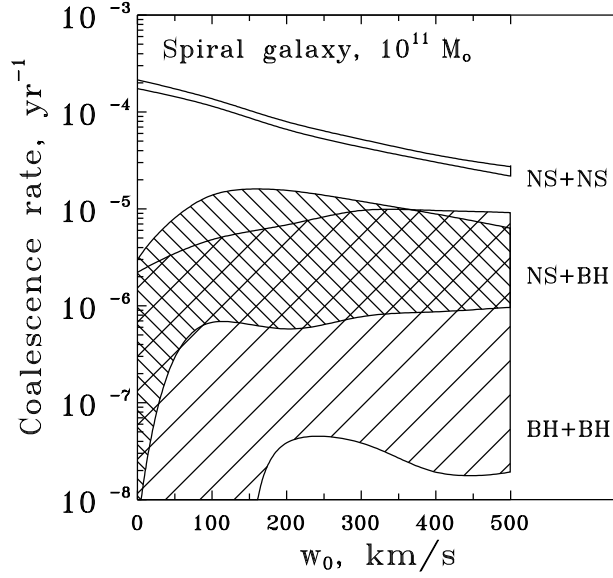
Figure 1. NS+NS, BH+NS, and BH+BH merging rates in a $10^{11} M_\odot$ galaxy as functions of the kick velocity parameter w_0 for the Lyne-Lorimer kick velocity distribution.

baryon content of the Universe, and arrived at the relationship

$$\mathcal{R}_V = 0.1 \mathcal{R}_G \left(\frac{r}{1 \; Mpc} \right)^3 .$$

Thus, within the volume of radius $r = 100 \; Mpc$, and during 1 year, one expects that there will be 3 of the NS+NS coalescences and only 0.3 of the NS+BH or BH+BH coalescences. The increase of the radius r to $r = 200 \; Mpc$ increases the volume and the event rates by the factor 8.

The calculated event rates and gravitational wave amplitudes emitted by individual binaries should be compared with the instrumental sensitivities. The important fact is that the mass of a typical neutron star is 1.4 $M_\odot$, whereas the mass of a typical stellar black hole is (10 - 15) $M_\odot$. Thus, a pair of black holes is a more powerful source of gravitational waves than a pair of neutron stars. The black hole binaries are less numerous than the neutron star binaries, but they can be seen, by a given instrument, from much larger distances. The observationally required situation is when the
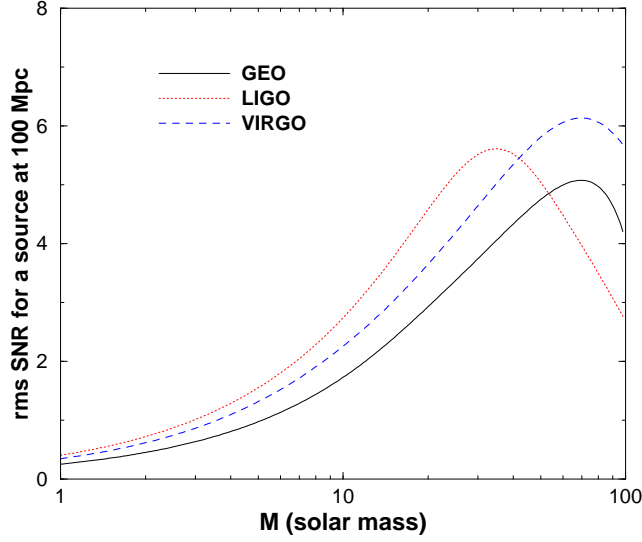
Figure 2. Signal-to-noise ratio, in initial interferometers, as a function of total mass, for inspiral signals from binaries of equal masses at 100 Mpc and averaged over source inclination.

detector's sensitivity allows one to see a few events per year, and at the signal to noise ratio S/N level of, at least, 2. Thus, the problem now is to find out the S/N for binaries of a given total mass M.

In Fig. 2 we plot [6] the (S/N) as a function of M. We have used the sensitivity curves of the initial interferometers in LIGO, VIRGO and GEO. It is assumed that the binaries consist of equal mass components, so the total mass of a NS+NS system is around $2.8 M_\odot$, while the total mass of a BH+BH system is around $(20 - 30) M_\odot$. The S/N curves are shown for binaries placed at $r = 100 \; Mpc$. The increase of the distance to $r = 200 \; Mpc$ decreases the values of the plotted functions by the factor 2.

It is seen from this graph that the S/N is less then 1 for the NS+NS binaries. Although the required 3 mergers of neutron stars do occur in the volume with the radius $r = 100 \; Mpc$, these sources cannot be regarded as detectable by the initial interferometers. On the other hand, for a pair of black holes with the total mass $M = (20 - 30) M_\odot$, the S/N can be at the level of 4, or even larger (in the LIGO detectors). The number of the BH+BH events in the discussed volume is too small, though. However, the S/N is still

at the level of 2, or better than 2, if the binary black hole is placed at the twice as large distance, i.e. $r = 200 \ Mpc$. As was shown above, one can expect about 2 - 3 BH+BH events in this larger volume. This is why we believe [6] that the coalescing black holes will probably be the first sources detected by the initial laser interferometers. It appears that other possible sources of gravitational waves are not likely to be detected at all by these instruments. The situation will become much better, and for all known sources, when the advanced interferometers become operational. And, of course, there exists a nonzero probability that the first detected signals will be totally unexpected to everyone, or something what the authors of Ref. [6] did not even include in their list of options. For additional information on astrophysical sources of gravitational waves and their detection, see the comprehensive contributions by Ferrari [8] and Vallisneri [9].

Relic gravitational waves are not expected to be accessible to the initial laser interferometers. The prospects are much better, though, for the advanced ground-based instruments and for the space-based interferometer LISA [10]. At very large scales, relic gravitational waves should also manifest themselves through the anisotropy and polarisation of the cosmic microwave background radiation (CMBR). It is fundamentally important to discover and study relic gravitational waves. They are the only clue to the processes that have taken place in the extreme conditions of the very early Universe and, indeed, they are the only clue to the origin of the Universe itself. Without observing and studying relic gravitational waves, there exists hardly any other way to test fundamental physical theories, such as superstrings and M-theory. The understanding that the expansion rate of the very early Universe could be grossly different from the one governed by the radiation-dominated fluid exists for a long time, at least, from the study of Sakharov [11]. In the recent years, however, there exists the unjust tendency to replace the notion and all the complexities of the very early Universe by a single word - "inflation". The reminder of my contribution is devoted to relic gravitational waves and the associated issues of cosmological perturbations of quantum-mechanical origin.

2   The inevitability of relic gravitational waves

The theory of almost all astrophysical sources of gravitational waves is based on classical physics. In contrast, the theory of relic gravitational waves involves the elements of quantum physics. It is the power and beauty of relic gravitational waves that their existence relies only on general relativity and basic principles of quantum field theory, such as the concept of zero-point quantum oscillations. To understand the generation of relic gravitons it is

convenient to start from a classical picture. Imagine a classical gravitational wave propagating in a nonstationary universe. The nonlinear character of the Einstein equations provides the coupling of the gravitational wave field to the smooth gravitational field of the evolving universe. The gravitational wave itself can be arbitrarily weak, so that the quadratic terms of the wave field can be neglected. However, the nonlinearity of the Einstein equations leads to the presence of the time-dependent background gravitational field in the coefficients of the wave equation. The background gravitational field plays the role of the pump field capable of increasing the amplitude of the propagating gravitational wave [12]. In fact, the traveling wave is being amplified, with the accompanying appearance of the wave traveling in the opposite direction. Together, they form an (almost) standing wave. This type of coupling of a wave-field to gravity is not something that is true for any wave-field. On the contrary, it is quite unusual. For instance, it does not take place for electromagnetic waves.

The quantum element enters the picture when we realise that if the initial gravitational waves represent only the inevitable zero-point quantum oscillations, the amplification mechanism should be at work. In precise technical terms, the quantum-mechanical Schrödinger evolution transforms the initial vacuum state of gravitational waves into a multi-quantum state known as squeezed vacuum quantum state [13]. It is the variance of phase that gets strongly diminished (squeezed) while the mean number of quanta and its variance get strongly increased. Of course, for this effect to be numerically considerable, the coupling of the wave field to the pump field should be considerable. The coupling is regulated by the ratio of the wave period to the characteristic time-scale of variations of the background gravitational field. If the pump field varies very slowly (adiabatically), this ratio is very small, and practically nothing happens to the wave. In other words, during the adiabatic regime, the vacuum state remains the vacuum state. In the opposite (superadiabatic) regime, the relative increase (as compared with adiabatic behaviour) of the wave amplitude, and the increase of the mean number of particles, is most dramatic. In the cosmological setting, the amplification mechanism is effective during the interval of time when the wavelength is comparable with, and longer than, the cosmological Hubble radius. Thus, the generating process (superadiabatic (parametric) amplification of the zero-point quantum oscillations [12]) is a local and dynamical process, it does not rely on the existence of globally defined event horizons of the background space-time. Moreover, in cosmological models of practical interest, there is no horizons at all. It is also irrelevant which particular matter source drives the gravitational pump field, be it a scalar field or something else. In some of its basic aspects, the

quantum-mechanical generation of cosmological perturbations is similar to the laboratory-tested generation of squeezed light.

The gravitational field of a homogeneous isotropic universe is given by the metric

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)g_{ij}\mathrm{d}x^i\mathrm{d}x^j = a^2(\eta)[-\mathrm{d}\eta^2 + g_{ij}\mathrm{d}x^i\mathrm{d}x^j] \ , \qquad (1)$$

where the scale factor $a(t)$ (or $a(\eta)$) is driven by a matter distribution with some effective (in general, time-dependent) equation of state. The perturbed gravitational field can be written (for simplicity, in a spatially-flat universe) as

$$\mathrm{d}s^2 = a^2(\eta)[-\mathrm{d}\eta^2 + (\delta_{ij} + h_{ij})\mathrm{d}x^i\mathrm{d}x^j], \qquad (2)$$

$$h_{ij}(\eta, \mathbf{x}) = \frac{\mathcal{C}}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d^3\mathbf{n} \sum_{s=1,2} \overset{s}{p}_{ij}(\mathbf{n}) \frac{1}{\sqrt{2n}} \left[\overset{s}{h}_n(\eta)e^{i\mathbf{n}\cdot\mathbf{x}} \overset{s}{c}_{\mathbf{n}} + \overset{s}{h}{}^*_n(\eta)e^{-i\mathbf{n}\cdot\mathbf{x}} \overset{s\dagger}{c}_{\mathbf{n}} \right] (3)$$

By expanding the functions $h_{ij}(\eta, \mathbf{x})$ over spatial Fourier harmonics $e^{\pm i n \mathbf{x}}$ we reduce the perturbed dynamical problem to the evolution of mode functions $\overset{s}{h}_n(\eta)$ for each mode $\mathbf{n}$. Two polarisation tensors $\overset{s}{p}_{ij}(\mathbf{n})$, $s = 1, 2$ have different forms depending on whether they represent gravitational waves, rotational perturbations, or density perturbations. In the case of gravitational waves, the polarisation tensors describe two polarisation states which are often called the "plus" and "cross" polarisations.

For the quantized field, the quantities $\overset{s}{c}_{\mathbf{n}}$, $\overset{s\dagger}{c}_{\mathbf{n}}$ are annihilation and creation operators satisfying the conditions

$$[\overset{s'}{c}_{\mathbf{n}}, \ \overset{s\dagger}{c}_{\mathbf{m}}] = \delta_{s's}\delta^3(\mathbf{n} - \mathbf{m}) \ , \quad \overset{s}{c}_{\mathbf{n}}|0\rangle = 0 \ , \qquad (4)$$

where $|0\rangle$ (for each $\mathbf{n}$ and s) is the fixed initial vacuum state defined at some η_0 in the very distant past, long before the superadiabatic regime for the given mode has started. In that early era, the mode functions $\overset{s}{h}_n(\eta)$ behaved as $\propto e^{-in\eta}$, so that each mode $\mathbf{n}$ represented a strict traveling wave propagating in the direction of $\mathbf{n}$. In the case of gravitational waves, the normalization constant $\mathcal{C}$ is $\sqrt{16\pi}l_{Pl}$. For cosmological density perturbations, which we will also discuss below, the normalisation constant is $\sqrt{24\pi}l_{Pl}$.

The calculation of quantum-mechanical expectation values and correlation functions provides the link between quantum mechanics and macroscopic physics. Using the representation (3) and definitions above, one finds the

variance of metric perturbations:

$$\langle 0|h_{ij}(\eta,\mathbf{x})h^{ij}(\eta,\mathbf{x})|0\rangle = \frac{\mathcal{C}^2}{2\pi^2}\int_0^\infty n^2 \sum_{s=1,2} |\overset{s}{h}_n(\eta)|^2 \frac{\mathrm{d}n}{n}. \tag{5}$$

The quantity

$$h^2(n,\eta) = \frac{\mathcal{C}^2}{2\pi^2}n^2 \sum_{s=1,2} |\overset{s}{h}_n(\eta)|^2 \tag{6}$$

gives the mean-square value of the metric (gravitational field) perturbations in a logarithmic interval of n and is called the (dimensionless) power spectrum. The power spectrum of metric perturbations is a quantity of great observational importance. It defines the temporal structure and amplitudes of the g.w. signal in the frequency bands of direct experimental searches. It is also crucial for calculations of anisotropy and polarisation induced in CMBR by relic gravitational waves and by other metric perturations. The inevitable squeezing makes the spectrum an oscillatory function of time, which can also be thought of as a consequence of the standing-wave pattern of the generated field. At every fixed moment of time, the spectrum, as a function of the wave-number n, has many maxima and zeros.

To find the power spectrum at any given moment of time (for instance, today or at the moment of decoupling of CMBR from the rest of matter) we need to know the mode functions at those moments of time. The mode functions are governed by the Heisenberg equations of motion whose interaction Hamiltonian participates with the coupling function proportional to a'/a. These equations are, of course, equivalent to the perturbed Einstein equations. The second-order "master equation" describing gravitational waves is [12]

$$\overset{s}{\mu}_n'' + \overset{s}{\mu}_n\left[n^2 - \frac{a''}{a}\right] = 0 , \tag{7}$$

where the functions $\overset{s}{\mu}_n(\eta)$ are related to the mode functions $\overset{s}{h}_n(\eta)$ by

$$\overset{s}{\mu}_n(\eta) \equiv a(\eta)\overset{s}{h}_n(\eta). \tag{8}$$

One can view Eq. (7) as the equation for a parametrically disturbed oscillator (the term in brackets is the square of its variable frequency), or as the Schrödinger equation for a particle moving in the presence of a potential barrier $W(\eta) = a''/a$ (while remembering that η is a time coordinate rather than a spatial coordinate).

As soon as the pump field (represented by $a(\eta)$) is known, and since the initial conditions are fully determined, the mode functions are strictly calculable. We know relatively well the behaviour of $a(\eta)$ at the matter-diminated (m) and radiation-dominated (e) stages of expansion. However, we do not know $a(\eta)$ at the most important preceeding stage of evolution, which we call the initial (i) stage. Ideally, if the full evolution of $a(\eta)$ had followed from some fundamental physical theory, the relic g.w. background would be uniquely determined, and we would have to live with what the fundamental theory dictates. In the absence of such a theory, we can only do calculations for various possible models and compare the results with observations. Conversely, the observational restrictions on relic gravitons, or their direct detection, is our clue to the dynamics of the very early Universe.

3 Calculating the power spectrum

The scale factor $a(\eta)$ at the matter-dominated stage, governed by whatever matter with the effective equation of state $p = 0$, behaves as $a(\eta) \propto \eta^2$. It is convenient to write $a(\eta)$ in the explicit form

$$a(\eta) = 2l_H(\eta - \eta_m)^2 \ , \qquad (9)$$

where l_H is the Hubble radius today and η_m is a constant explained below. The moment of time "today" (in cosmological sense) is labeled by $\eta = \eta_R$ (the subscript R denoting "reception"). It is convenient to choose

$$\eta_R - \eta_m = 1. \qquad (10)$$

With this convention, $a(\eta_R) = 2l_H$, and the wave, of any physical nature, whose wavelength λ today is equal to today's Hubble radius, carries the constant wavenumber $n_H = 4\pi$. Longer waves have smaller n's and shorter waves have larger n's, according to the relationship $n = 4\pi l_H/\lambda$. For example, the ground-based gravitational wave detectors are most sensitive to frequencies around $30Hz - 3000Hz$. The corresponding wavelengths have wavenumbers n somewhere in the interval $10^{20} - 10^{22}$.

The matter-dominated era was preceded by the radiation-dominated era with the scale factor $a(\eta) \propto \eta$. Without any essential loss of generality, we assume that the transition from e era to m era was instantaneous and took place at some $\eta = \eta_2$. The redshift of the transition is z_{eq}: $a(\eta_R)/a(\eta_2) = 1 + z_{eq}$. It is believed that z_{eq} is somewhere near 6×10^3. In its turn, the radiation-dominated era was preceded by the initial era of expansion, whose nature and scale factor are, strictly speaking, unknown. To simplify the analysis, and

since the wave equations admit simple exact solutions in the case of power-law scale factors, we assume that the i era, similar to the e and m eras, was also described by a power-law scale factor. Following the early notations [12], we parametrize the i era by $a(\eta) \propto |\eta|^{1+\beta}$. The transition from i era to e era takes place at some $\eta = \eta_1$ and at redshift z_i: $a(\eta_R)/a(\eta_1) = 1 + z_i$. Further analysis shows (see below) that in order to get the right amplitude of the generated perturbations, the numerical value of z_i should be somewhere near 10^{30}. Whether the i era was governed by a scalar field φ (the central element of inflationary scenaria) or by something else, is irrelevant. What is relevant is the functional form of the gravitational pump field represented by the scale factor $a(\eta)$.

We now write the full evolution of the growing scale factor explicitly:

$$a(\eta) = l_0 |\eta|^{1+\beta}, \quad \eta \le \eta_1, \quad \eta_1 < 0, \quad \beta < -1 \ , \tag{11}$$

$$a(\eta) = l_0 a_e (\eta - \eta_e), \quad \eta_1 \le \eta \le \eta_2 \ , \tag{12}$$

$$a(\eta) = 2 l_H (\eta - \eta_m)^2, \quad \eta_2 \le \eta. \tag{13}$$

The continuous matching of $a(\eta)$ and $a'(\eta)$ at the transition points determines all the participating constants in terms of l_H, z_i, z_{eq} and β. Thus, we are essentially left with the two unknown parametrs: z_i and β. The z_i is primarily responsible for the amplitude of the generated perturbations, while the β is responsible for the spectral slope of the metric power spectrum. If $\beta = -2$, the generated spectrum is flat, that is, independent of the wave-number n. The flat spectrum is also called the Harrison-Zeldovich-Peebles spectrum and is characterised by the spectral index n $= 1$. The relationship between the spectral index n and parameter β is n $= 2\beta + 5$. Inflationary models governed by scalar fields φ are uncapable of producing the "blue" power spectra, i.e. spectra with n > 1 ($\beta > -2$). Indeed, $\beta > -2$ requires the effective equation of state at the i stage to be $\epsilon + p < 0$, but this cannot be accomodated by any scalar field whatever the scalar field potential $V(\varphi)$ may be [6].

For the CMBR calculations one also needs the redshift z_{dec} of the last scattering surface $\eta = \eta_E$ (with the subscript E denoting "emission"), where the CMBR photons have decoupled from rest of the matter: $a(\eta_R)/a(\eta_E) = 1 + z_{dec}$. The numerical value of z_{dec} is somewhere near 1000.

Since the scale factor and initial conditions for the mode functions have been strictly defined, the power spectrum (6) is unambiguously calculable. We will present the results of calculation of $h^2(n, \eta)$ taken at the time of
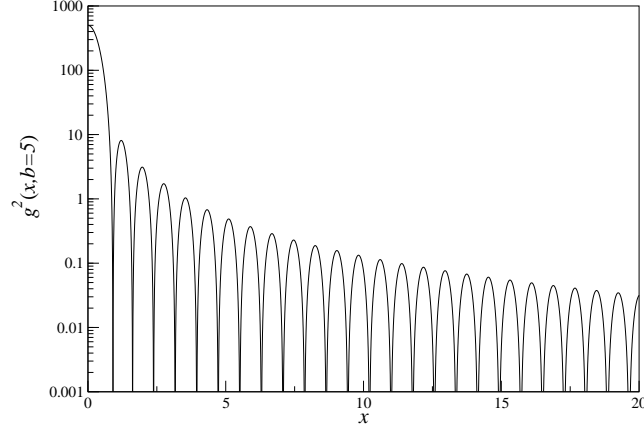
Figure 3. Plot of $g^2(x, b = 5)$ versus x.

decoupling $\eta = \eta_E$. The exact form of the power spectrum is

$$h^2(n, \eta_E) = \frac{l_{Pl}^2}{4\pi l_H^2}(1 + z_{dec})n^4|B|^2 g^2(x, b) \quad , \tag{14}$$

where

$$g^2(x, b) = [\rho_1(x)j_1(bx) - \rho_2(x)j_{-2}(bx)]^2 \,, \quad x \equiv \frac{n}{2\sqrt{1 + z_{eq}}}, \quad b \equiv \frac{2\sqrt{1 + z_{eq}}}{\sqrt{1 + z_{dec}}} \,.$$

$$\rho_1(x) = \frac{1}{x^2} \left[(8x^2 - 1)\sin x + 4x \cos x + \sin 3x \right] \,,$$

$$\rho_2(x) = \frac{1}{x^2} \left[(8x^2 - 1)\cos x - 4x \sin x + \cos 3x \right] \,,$$

and $j_1(bx)$, $j_{-2}(bx)$ are spherical Bessel functions. The quantity $n^4|B|^2$ is, in general, n-dependent and β-dependent, but for the case of $\beta = -2$ it simplifies to

$$n^4|B|^2 = \frac{4(1 + z_i)^4}{(1 + z_{eq})^2}, \qquad \beta = -2. \tag{15}$$

In Fig. 3 we show [14] the function $g^2(x, b)$ for $b = 5$.

4   Detectability of relic gravitational waves

We start from a discussion of manifestations of relic gravitational waves in the CMBR anisotropies. It can be shown (see [14] and references there) that the angular correlation function has the universal form

$$\left\langle 0 \left| \frac{\delta T}{T}(\mathbf{e}_1) \frac{\delta T}{T}(\mathbf{e}_2) \right| 0 \right\rangle = \sum_{l=2}^{\infty} \frac{2l+1}{4\pi} C_l P_l(\cos \delta), \tag{16}$$

where $P_l(\cos \delta)$ are Legendre polynomials for the separation angle δ between the unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$, and the multipole moments C_l are explicitly determined by the metric mode functions.

In Fig. 4 we show [14] by a solid line the graph of the function $l(l+1)C_l$ calculated for the g.w. background with the power spectrum (14). The cosmological parameters were taken, for illustration, as $z_{eq} = 10^4$, $z_{dec} = 10^3$, $\beta = -2$. The parameter z_i is adjusted in such a manner ($z_i = 10^{29.5}$) that the graph goes through the point $l(l+1)C_l = 6.4 \times 10^{-10}$ at $l = 10$, which agrees with observations. For comparison, the dashed line shows the same function, but calculated for the alternative (non-squeezed) background of gravitational waves of the same averaged power density. The remarkable (even if expected) result is that the background of non-squeezed (traveling) gravitational waves does not produce oscillations in the angular power spectrum C_l, whereas the background of squeezed (standing) gravitational waves does. The peaks and dips of the angular power spectrum are in close relationship with the maxima and zeros of the metric power spectrum of Fig. 3. It is unlikely that the peaks and dips caused by relic gravitational waves can be observationally revealed, but this calculation serves as a guidance for the related explanation (see below) of the actually observed peaks and dips [15] that lay at a higher level of the signal.

For purposes of direct experimental searches for relic gravitational waves, one needs to know the metric power spectrum in the present era. In Fig. 5 we show the piece-wise envelope of the r.m.s. quantity $h(n, \eta_R)$ plotted as a function of frequency ν in $Hertz$.

This particular graph is derived under the assumption that the parameter β is $\beta = -1.9$, which corresponds to the spectral index n = 1.2. A few years ago (see, for example, [16]), this value of n was quoted as the one that follows from the lower-order multipoles of the CMBR data. The most recent analyses, which include data on the large scale structure, seem to prefer a smaller value of n. Although the often cited central values of n are still somewhat larger than 1, the 1σ error-bars do not usually include the value n = 1.2. One should remember, however, that the current analyses of the data are consistent only if
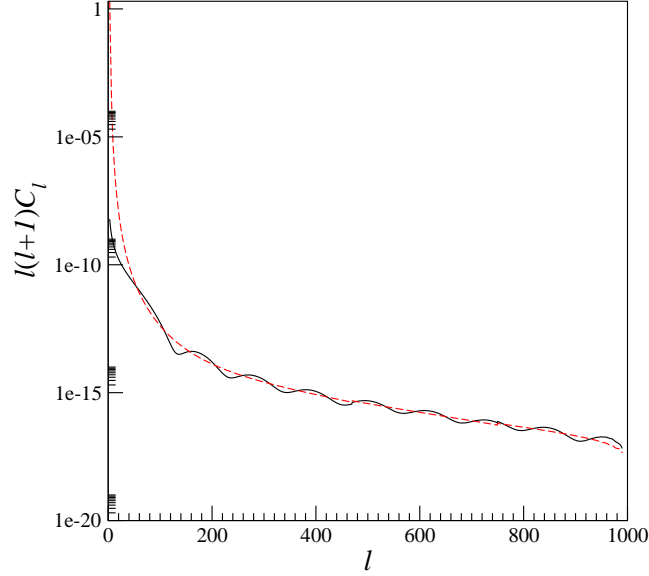
Figure 4. The solid line depicts the plot of $l(l+1)C_l$ versus l, normalized such that at $l = 10$, we have $l(l+1)C_l = 6.4 \times 10^{-10}$, which tallies with observations. The dashed line is the corresponding plot for the non-squeezed g.w. background. We take $\beta = -2$, and the redshifts $z_{eq} = 10^4$ and $z_{dec} = 10^3$.

one assumes that about 70 percent of "matter" in the present Universe resides in the cosmological Λ term or in a "dark energy/quintessence", which is not subject to gravitational clustering. It is still not clear whether this conclusion is one of the greatest discoveries about the Universe, or it may be a result of improper evaluation of the role of metric perturbations in the CMBR analysis. We will comment on this issue in the next section. In addition, one should bear in mind that the flat (n = 1) and "red" (n < 1) spectra possess a theoretical difficulty: the integral over wavenumbers n for the mean square value of the metric (gravitational field) perturbations diverges at the lower limit $n \to 0$, i.e. for very long waves. The removal of these divergencies would require some extra assumptions. In any case, we will discuss the prospects of direct detection of relic gravitational waves assuming that the graph in Fig. 5 is correct or close to correct.

According to analytical formulas and Fig. 5, the theoretically expected r.m.s. value at $\nu = 100 Hz$ is $h_{th} = 10^{-25}$. The target value of the LIGO-II [17]
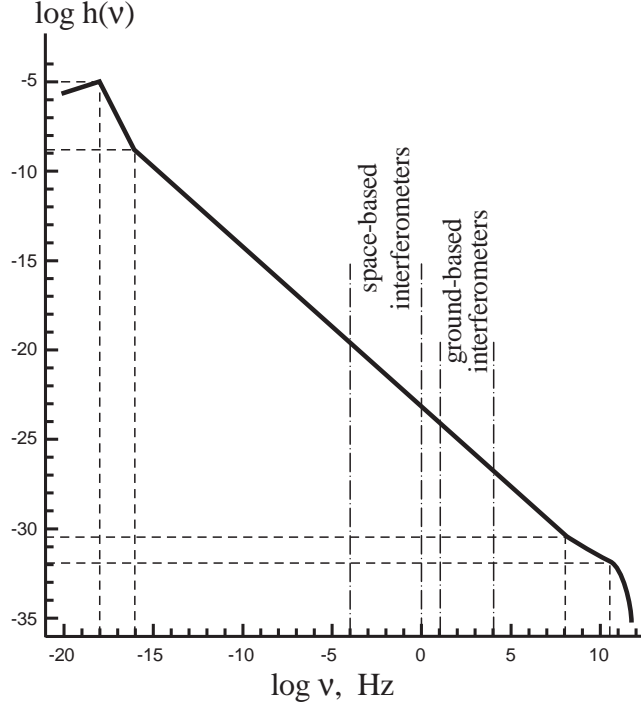
Figure 5. Expected spectrum $h(\nu)$ for the case $\beta = -1.9$.

at this frequency is $h_{ex} = 10^{-23}$. The remaining gap of 2 orders of magnitude can be covered by the cross-correlation of sufficiently long strips of data from two or more detectors. The S/N ratio will be better than 1, if the common integration time exceeds $\tau = 10^6 sec$. This does not look like a hopeless task. The situation is somewhat better in the LISA frequency interval. According to evaluations of Ref. [6], a 1-year observation time can provide a S/N ratio around 3 in the frequency interval $(2 \times 10^{-3} - 10^{-2})Hz$. These estimates do not include a possible improvement in the S/N, which can be achieved if one succeeds in finding a sophisticated method of exploiting, at these frequencies, the ever-present squeezing. This improvement would certainly be possible in a very narrow-band detector, but it is unclear whether it can ever be realised in a realistic broad-band detector (for a discussion, see [18] and [6]). At the present time, it is difficult to say at which scales the relic gravitational waves

will be first discovered. This can happen in direct experiments at relatively small scales, or, indirectly, at cosmological scales, through the anisotropy and polarisation measurements of the CMBR.

5   Relic gravitational waves and primordial density perturbations

The superadiabatic (parametric) mechanism is always operative for gravitational waves, but its applicabilty to density perturbations requires special assumptions about properties of matter. Of course, the notion of density perturbations includes the associated metric perturbations h_{ij} of Eq. 3; the matter and gravitational field (metric) variables are always linked by the perturbed Einstein equations. If the i stage was governed by a scalar field φ coupled to gravity in a special manner (a hypothesis usually assumed in inflationary scenaria), the quantum-mechanical generation of primordial density perturbations, in addition to the inevitable generation of relic gravitational waves, becomes possible. The underlying physical theory for these two respective fields is exactly the same, and therefore the general properties of relic gravitational waves and primordial density perturbations (squeezing, standing-wave pattern, spectral slopes, numerical values of the amplitudes, etc.) are almost identical. In particular, relic gravitational waves and primordial density perturbations should provide approximately equal contributions to the lower-order multipoles (starting with the quadrupole moment C_2) of the CMBR anisotropies [19]. Some differences arise, however, at the late stage of their evolution. In the matter-dominated era, and for wavelengths comfortably shorter than the Hubble radius, numerical values of the metric components associated with density perturbations grow in proportion to $a(\eta)$ (gravitational instability), whereas the amplitudes of gravitational waves decrease in proportion to $1/a(\eta)$ (adiabatic behaviour). This explains why the metric amplitudes of these two fields, being of the same order of magnitude at scales comparable with the prersent-day Hubble radius, are significantly different at much smaller scales. This difference makes gravitational waves subdominant in their contribution to the CMBR multipoles near the peak at $l \sim 200$, and makes them so small and difficult to detect at laboratory scales.

As a benchmark for the spectral index one can take $\beta = -2$, i.e. flat spectrum. The CMBR anisotropies caused by gravitational waves are shown in Fig. 4. The lowest excited multipole is $l = 2$ (quadrupole). In contrast, density perturbations produce all multipoles, including $l = 0$ (monopole) and $l = 1$ (dipole). In general, CMBR anisotropies caused by density perturbations have contributions from 3 different ingredients (for a recent detailed

discussion, see [20]). Specifically, the ingredients are: the accompanying metric perturbations (mostly driven by the gravitationally dominant matter, which is, according to the existing views, a presureless cold dark matter (CDM)), the intrinsic temperature variations at the last scattering surface, and the velocities (with respect to the coordianate system comoving with the CDM) of the last scattering electrons.

The angular power spectrum $l(l+1)C_l$ is usually presented as an outcome of numerical calculations, where all contributions are being mixed up. However, the evolutionary equations, on which the numerical codes are based, allow analytical calculations for the lower multipoles, i.e. for l's up to $l \approx (30 - 40)$. It is often claimed that the analytical limit of the employed evolutionary equations produces the lower-order multipoles C_l according to the formula

$$C_l = \frac{const.}{l(l+1)}.$$ (17)

This formula predicts an infinitely large monopole moment C_0, and the dipole moment C_1 only a factor 3 greater than the quadrupole moment C_2; both predictions are in a severe conflict with observations. Moreover, this formula misses the quite steep growth, beginning from $l = 2$, of the function $l(l+1)C_l$, which must take place for purely gravitational reasons, that is, when the metric perturbations are treated correctly [21].

It was demonstrated [21] that formula 17 is incorrect and cannot be the outcome of correct equations. Specifically, formula 17 arises only if one unjustifiably neglects 3 out of 4 terms in the exact formula (43) of the Sachs-Wolfe paper [22]. The correct handling of metric perturbations makes the monopole moment C_0 finite and small, whereas the dipole moment C_1 becomes 5 orders of magnitude greater than the number following from formula 17, and, thus, the theoretical (statistical) C_1 comes in agreement with the observed value of C_1. It was also shown that the growth of the function $l(l + 1)C_l$ would be unlimited, if one were allowed to extrapolate the flat spectrum to arbitrarily large wave-numbers n. This is illustrated in Fig 6.

In reality, however, the modulating (transfer) function of the metric perturbations bends the angular spectrum down and introduces a series of peaks and dips, analogous to the peaks and dips caused by gravitational waves in Fig. 4. We will discuss peaks and dips shortly, but first we shall say a few words about the importance of theoretical calculations of the expected (statistical) value of the dipole moment C_1. Because of statistical properties of the generated perturbations, all statistical multipoles C_l are independent of the observer's position.
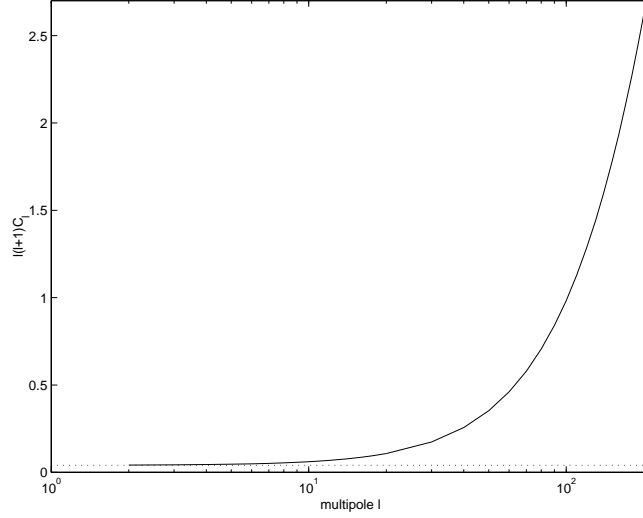
Figure 6. The multipole distributions as described by the exact formula (solid line) and formula 17 (dotted line) for the flat primordial spectrum $\beta = -2$ (n = 1).

The derivation of formula 17 takes into account the perturbations of all wavelengths. The integral over n, which gives rise to this formula, is formally extended from $n = 0$ to arbitrarily large n's. If this formula were correct, it would be applicable all over the linear regime of cosmological perturbations and could possibly break down only due to nonlinearities at relatively short scales. The gap in 5 orders of magnitude between the statistical prediction of this formula and the actually observed C_1 would need to be attributed either to extremely improbable realisation of the perturbed gravitational field, or to the quite local conditions where nonlinearities set it and where the linear approximation is not valid. In the latter case, numerical values of C_1's observed by inhabitants of the not-too-distant galaxies would not need to have anything in common with each other and with the C_1 observed by us. In contrast, calculations of Ref. [21] show that numerical value of the statistical C_1 is most sensitive to scales where the metric power spectrum bends down. In other words, the corresponding integral is primarily accumulated at scales of the order of $(100 - 200)Mpc$, i.e. well within the linear regime. This result comes about from the "gradient of gravitational potential" terms (see also [14]), which are totally neglected in the derivation of formula 17. This means that the galaxies within, at least, the radius of $(100 - 200)Mpc$, and, at least, the

field galaxies, are expected to have approximately equal observed C_1's. This number should also be of approximately the same value as our own C_1 (as soon as our measured C_1 turns out to be close to the statistical C_1). In principle, this picture is observationally testable. Of course, we cannot place ourselves at different galaxies and measure the C_1 there. But, it is not excluded that the CMBR dipoles seen at other galaxies can be evaluated indirectly, possibly with the help of the Sunyaev-Zeldovich effect. It would be very interesting if these observational evaluations could be done. In any case, the calculation of the theoretical (statistical) C_1 is as important as the calculation of other multipoles. It would be embarassing if some of currently popular cosmological models would predict the statistical C_1 way out of what is actually observed by us and by observers in other galaxies (assuming that we can evaluate their dipoles).

It is argued in Ref. [14] that the (squeezed) metric perturbations associated with density perturbations are mostly responsible for the recently observed peaks and dips in the angular power spectrum $l(l+1)C_l$. This interpretation is different from the concept of "acoustic peaks" based on the existence of the plasma sound waves at the last scattering surface. The rigorous evolution of the quantum-mechanically generated density perturbations through the radiation-dominated and matter-dominated eras, allows one to find the metric power spectrum, defined by Eq. 6, at the last scattering surface $\eta = \eta_E$. The analytical formula for $h^2(n, \eta_E)$ is given by

$$h^2(n, \eta_E) = \frac{\mathcal{C}^2}{2\pi^2} \frac{n^4 |B|^2 (1 + z_{eq})}{48 l_H^2} \left(\frac{\sin y_2}{y_2} \right)^2 \frac{(300 - 20 p^2 y_2^2 + p^4 y_2^4)}{200}, \quad (18)$$

where

$$y_2 = \frac{n}{2\sqrt{3}\sqrt{1 + z_{eq}}}, \quad \text{and} \quad p \equiv \frac{2\sqrt{3}\sqrt{1 + z_{eq}}}{\sqrt{1 + z_{dec}}}.$$

The graph of the oscillating function

$$f^2(x, p) \equiv \left(\frac{\sin x}{x} \right)^2 \left[300 - 20 p^2 x^2 + p^4 x^4 \right],$$

where $x \equiv y_2$ and, for illustration, we take $p = 8$, is shown in Fig. 7.

The oscillations in the metric power spectrum are a direct consequence of squeezing (standing-wave pattern) of the primordial density perturbations. These oscillations find their reflection in the peaks and dips of the angular power spectrum. The main difference with the concept of "acoustic peaks" lays in the two facts: the amplitudes of metric perturbations are larger than the amplitudes of matter density variations in the photon-electron-baryon
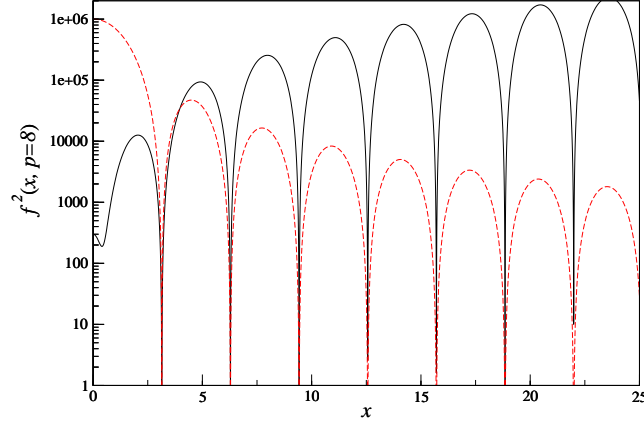
Figure 7. The plot depicted by the solid line is that of $f^2(x, p = 8)$ versus x. The dashed line shows the behavior of the function $(\sin x/x)^2 \times 10^6$.

fluid, and the wave-number periodicity of metric perturbations is governed by a sine function, instead of a cosine function for acoustic perurbations. It is argued [14] that the actually observed features are better described by the derived analytical formulas than by the concept of "acoustic peaks".

The study of density perturbations is directly relevant to the problem of relic gravitational waves. If cosmological perturbations are indeed generated quantum-mechanically (what the present author believes is likely to be true), many properties of primordial density perturbations and relic gravitational waves should be common. By studying density perturbations (for instance, by proving the presence of squeezing) we will be more confident about the properties of relic gravitational waves.

One should be aware that the story of quantum-mechanically generated cosmological perturbations is dramatically different in inflationary scenario. Inflationists love to speak about "stretching the quantum fluctuations of the inflaton to cosmological scales". As a result of this "stretching", they manage to produce the arbitrarily large amplitudes of today's density perturbations. Indeed, the "standard result" of inflationary scenario predicts the infinitely large density perturbations, in the limit of the flat spectrum (n = 1), through the set of evaluations: $\delta\rho/\rho \sim h_S \sim \zeta \sim H^2/\dot\varphi \sim V^{3/2}(\varphi)/V'(\varphi) \sim 1/\sqrt{1 - \mathrm{n}}$. [The "quantization", as understood by inflationists, of the necessarily coupled system of scalar field and metric perturbations is such, that the so-called Bardeen's gauge-invariant quantity ζ is arbitrarily large already at the begin-

ning of the amplifying (superadiabatic) regime, and then, being "conserved", this arbitrarily large number translates to the end of the amplifying regime, and to the present time.] Of course, this "standard result" is in full disagreement not only with the theoretical quantum mechanics, but with available observations too, as long as the error-boxes of the observationally derived spectral index n are centered at $n \approx 1$ and include the value $n = 1$. However, by composing the ratio of the gravitational wave amplitude h_T to the predicted divergent amplitude of the scalar metric perturbations h_S (the so called "consistency relation": $h_T/h_S \approx \sqrt{1-n}$), inflationary theorists substitute their prediction of arbitrarily large density perturbations for the claim that it is the amount of gravitational waves that should be zero, or almost zero, at cosmological scales and, hence, down to laboratory scales. This claim has led to many years of mistreatment of a possible g.w. contribution to the CMBR data. The "standard inflationary result" is maintained by inflationists and their followers until now. For instance, the recent review paper [23] claims that the initial spectrum of gravitational waves is "constrained to be small compared with the initial density spectrum". It is only in a few recent papers (for example, [24]) that the inflationary "consistency relation" is not being used when analyzing the CMBR and large scale structure observations, with some interesting conclusions. The inflationary claim of the "negligible" amount of gravitational waves influences also the LISA community. It is sometimes opined that the LISA parameters should be better optimised for detection of g.w. noise from the multitude of binary white dwarfs, rather than for a search for relic gravitational waves. There seems to be some logic behind this opinion. If one hears so often about "excellent agreement" of inflationary predictions with observations, and if the central of these predictions is "negligible" amount of relic gravitational waves, there is no need to bother about them. Many of the contemporary writers manage to do both: to praise inflationary predictions and to speculate about "testing inflation" with the help of gravitational waves, including the CMBR polarisation measurements and post-LISA missions. Apparently, they plan to do this at the signal to noise ratio level of 10^{-20}, or so, for "inflation predicted" gravitational waves. For some reason, those authors do not suggest to test inflationary predictions right from the predicted divergent amplitudes of density perturbations. Sometimes, inflationary calculations based on the "standard result" and "consistency relation" arrive at gravitational-wave amplitudes which are only 1-2 orders of magnitude smaller than, or even comparable (for the spectral index n sufficiently smaller than 1) with the metric amplitudes of density perturbations. Of course, this does not make the fundamentally wrong "standard result" a correct one. One can find more reading about the "standard inflationary

result" and its critique, including the earlier claim of inflationists that their divergent amplitudes of density perturbations are justified by the (incorrect and now abandoned) concept of "big amplification during reheating", in the end of Sec. 6 of Ref. [6] and in references there.

6   Conclusions

It is important to understand that the existence of relic gavitational waves relies only on general relativity and basic principles of quantum field theory. The detection of relic gravitational waves will be a direct probe of the expansion rate of the very early Universe, and not a test of some extra assumptions, such as the dominance of a scalar field or the form of its potential $V(\varphi)$. If the observed large-scale CMBR anisotropy $\delta T/T$ is indeed caused by cosmological perturbations of quantum-mechanical origin, the contribution of relic gravitational waves to $\delta T/T$ must be considerable. The extrapolation of the g.w. metric power spectrum to shorter wavelengths shows that, depending on the still existent uncertainty in the spectral index, relic gravitational waves may be measurable by the advanced ground-based or space-based laser interferometers. The ever-present squeezing must manifest itself in the periodic structure of the metric power spectra and, at cosmological scales, in the oscillatory behaviour of the CMBR multipoles C_l as a function of l. When evaluating the scientific importance and prospects of discovery of relic gravitational waves, it is necessary to respect the conclusions of quantum mechanics and general relativity, and not the statements of popular fallacies.

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